Connect, Not Collapse: Explaining Contrastive Learning for Unsupervised Domain Adaptation



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Unsupervised domain adaptation (UDA)

Labeled source domain



Clock

Unsupervised domain adaptation (UDA)

Labeled source domain



Clock

Unlabeled target domain





Unsupervised domain adaptation (UDA)

Labeled source domain





?

Unlabeled target domain

Goal: high accuracy on target domain (without labels)

Labeled source domain



representations

Unlabeled target domain



Target representations

Labeled source domain



Unlabeled target domain

Target representations

Labeled source domain Unlabeled target domain



Motivated by theories such as $H\Delta H$ divergence (Ben-David et al 2010): want source and target reps to be "indistinguishable" to get good target accuracy

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Pre-training for UDA

Step 1: pre-train on unlabeled data (combined source + target)





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Step 2: fine-tune on labeled data (source)



Pre-training for UDA

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Step 2: fine-tune on labeled data (source)



Step 3: evaluate accuracy (target)

Inspired by e.g., Blitzer et al 2007

Contrastive pre-training (SwAV, Caron et al. 2020) is competitive with UDA methods (even when all methods use the same augmentations)



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80

Conventional hypothesis: does contrastive pre-training automatically merge the features across domains to achieve low $H\Delta H$ -divergence? SwAV + Extra: unlabeled Target Test 40 pre-training data = all 4 30 domains (DomainNet) or 20 all of ImageNet (Living-17, 10 Entity-30) 0 DomainNet Living-17 Entity-30 STL → CIFAR

ERM

Inspect DANN vs contrastive learning features: train discriminator between domains or between classes

Domain 1 (Sketch)

Class 1 (Butterfly)







Domain 2 (Real)





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Class 2 (Clock)



Between domains

Contrastive: 8% err



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Between domains DANN: 14% err Contrastive: 8% err



Inspect DANN vs contrastive learning features: train discriminator between domains or between classes

Domain 1 (Sketch)

Class 1 (Butterfly)



Domain 2 (Real)



Between classes DANN: 6% err Contrastive: 7% err

Class 2 (Clock)



Between domains DANN: 14% err Contrastive: 8% err



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Pre-training does not produce domain invariant features, and domains are about as "far apart" as classes!

• Performs competitively with strong baselines: SENTRY (Prabhu et al. 2021), DIRT-T (Shu et al. 2018), and DANN (Ganin et al. 2016)

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Why do these features still generalize to the target without domain invariance?

Outline

- Setup: augmentation graph
- Intuitions and theoretical results
 - Main intuitions (toy example)
 - Results for stochastic block model & beyond
 - Contrastive pre-training vs. ERM & DANN
- Test theoretical predictions on real data

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• Contrastive learning hinges on *positive pairs* (augmentations of the same original input)

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- Contrastive objective:
 - map positive pairs to similar features
 - map augmentations of different inputs to different features

Domain 1 (Sketch)



Class 1 (Butterfly)

Domain 2 (Real)



Class 2 (Clock)





Domain 1 (Sketch)

Domain 2 (Real)





 α : probability that augmentations of images coincide













Magnitudes of connectivity parameters ρ , α , β , and $\gamma \approx$ similarity of augmentations



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- Binary classification, 1 example per class and domain (4 examples total)
- Let \hat{F} : $R^{4 \times 3}$ be a matrix whose rows contain learned features



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Augmentation graph

If $\min(\alpha, \beta) > \gamma$ (and self-loop ρ is the largest):



Augmentation graph

If $\min(\alpha, \beta) > \gamma$ (and self-loop ρ is the largest):



Learned representation space

If $\min(\alpha, \beta) > \gamma$ (and self-loop ρ is the largest):











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Key condition for transfer: augmentations are more likely to change **only domain** (α) or **only class** (β) than **both domain and class** (γ)

If instead $\alpha < \gamma$:











If instead $\alpha < \gamma$:



If the condition is violated, the target features can be "swapped" so that a source-trained linear classifier fails to generalize

Generalization beyond simple example

• Consider stochastic block model (SBM): extends to multiple domains, multiple classes, and multiple examples per class/domain

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Generalization beyond simple example

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- We prove: same conditions $(\min(\alpha, \beta) > \gamma \text{ and } \rho \text{ is largest})$ allow contrastive pre-training to learn linearly transferable features (with easily separable source and target features)
- Follow-up work generalizes beyond random graph models, with asymmetry: HaoChen et al. 2022

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• Estimate w_1, w_2 by fitting a linear function in log space and determine quality of fit compared to a control

Predicting target accuracy (contrastive methods)



Predicting target accuracy (controls)



MoCo-V3

0.60

Predicting target accuracy (controls)



methods: DANN and SENTRY

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	$f_{ m src}$ vs. $f_{ m tgt}$	$f_{ m src}$ vs. $f_{ m dom}$	$f_{ m tgt}$ vs. $f_{ m dom}$
Living-17	0.397	0.013	0.016
DomainNet	0.187	0.018	0.018





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Orthogonal

Target Unlabeled Data is Important

 Access to target unlabeled examples is important for robustness (pretraining on source examples alone does not lead to robustness gains)

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 Access to target unlabeled examples is important for robustness (pretraining on source examples alone does not lead to robustness gains)

	ERM	SwAV (S)	SwAV (T)	SwAV (S+T)
Living-17	63.29	62.71	70.41	75.12
Entity-30	52.52	52.33	60.33	62.03

Concluding Thoughts: Why Pretraining?

• Rich organization can pretrain once, everyone can fine-tune for many tasks cheaply

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- Rich organization can pretrain once, everyone can fine-tune for many tasks cheaply
- This approach gets SoTA on many robustness datasets: WILDS-FMoW, WILDS-iWildCam, ImageNet robustness, DomainNet
- Our paper: why does pretraining help? Is it just about having lots of data?

Conclusion

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- Contrastive pre-training is a competitive method for UDA
- Works without collapsing source and target representations
- Instead, disentangles class and domain information, enabling transfer
 - Consequence of the structure of connections between domains and classes
 - via data augmentations

Subgroup Robustness Grows on Trees: An Empirical Baseline Investigation

IFDS Workshop on Distributional Robustness Aug. 5, 2022



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Subgroup Robustness

Subgroup Robustness refers to the ability of a model to achieve good performance across discrete subgroups in a distribution.

This is an extreme version of **subpopulation shift** where we evaluate shift on target datasets entirely of a single (demographic) subpopulation.





With Great Progress Comes Great...Confusion?

Rapid Progress In Robust Learning

- Maximum Weighted Loss Discrepancy (Khani et al. 2019)
- DRO (e.g. Duchi and Namkoong 2018, Levy et al. 2020)
- Group DRO (Sagawa et al. 2020)
- DORO (Zhai et al. 2021)
- ...many more!

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Need for Reliable Evaluation

In other fields, large-scale empirical baseline evaluations have been critical to (re)assessing progress (Liao et al. 2021).

The use of unreliable statistical inference methods in particular has led to misleading signals of progress (Agarwal et al. 2021).

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What is the current SOTA for subgroup robustness in tabular data?

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Two Perspectives on Subgroup Robustness

Study Design + Datasets

Results

Accuracy-Robustness Frontiers

Evaluating Evaluation Metrics + Model Selection Effects

Hyperparameter Sensitivity

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Subgroup Robustness and Fairness



Subgroup Robustness and Fairness



Tabular Data: Deep Learning's "Unconquered Castle"

Widely used in practice but often a secondary focus in evaluating robust models

Often **directly encodes** sensitive subgroups of interest

Challenging to model and SOTA performance is achieved with **non-neural methods**



[Sagawa et al. 2019]

Toxic	Comment Text	Male	Female	LGBTQ	White	Black	 Christian
0	I applaud your father. He was a good man! We need more like him.	1	0	0	0	0	 0
0	As a Christian, I will not be patronizing any of those businesses.	0	0	0	0	0	 1
0	What do Black and LGBT people have to do with bicycle licensing?	0	0	1	0	1	 0
0	Government agencies track down foreign baddies and protect law- abiding white citizens. How many shows does that describe?	0	0	0	1	0	 0
1	Maybe you should learn to write a coherent sentence so we can understand WTF your point is.	0	0	0	0	0	 0

[Koh et al. 2020]

Current tabular SOTA

Model/baseline for most robustness experiments

	HELOC		Adult		HIGGS		Covertype		Cal. Housing	
	Acc ↑	AUC ↑	Acc ↑	AUC ↑	Acc †	AUC ↑	Acc ↑	AUC ↑	MSE ↓	
Linear Model	73.0±0.0	80.1±0.1	82.5±0.2	85.4±0.2	64.1±0.0	68.4±0.0	72.4±0.0	92.8±0.0	0.528±0.008	
KNN [65]	72.2±0.0	79.0±0.1	83.2±0.2	87.5±0.2	62.3±0.1	67.1±0.0	70.2 ± 0.1	90.1±0.2	0.421 ± 0.009	
Decision Tree [197]	80.3±0.0	89.3±0.1	85.3±0.2	89.8±0.1	71.3±0.0	78.7±0.0	79.1±0.0	95.0±0.0	0.404 ± 0.007	
Random Forest [198]	82.1±0.2	90.0±0.2	86.1±0.2	91.7±0.2	71.9±0.0	79.7±0.0	78.1±0.1	96.1±0.0	0.272 ± 0.006	
XGBoost [53]	83.5±0.2	92.2±0.0	87.3±0.2	92.8±0.1	77.6±0.0	85.9±0.0	97.3±0.0	99.9±0.0	0.206 ± 0.005	
LightGBM [78]	83.5±0.1	92.3±0.0	87.4±0.2	92.9±0.1	77.1±0.0	85.5±0.0	93.5±0.0	99.7±0.0	0.195±0.005	
CatBoost [79]	83.6±0.3	92.4±0.1	87.2±0.2	92.8±0.1	77.5±0.0	85.8±0.0	96.4±0.0	99.8±0.0	0.196 ± 0.004	
Model Trees [199]	82.6±0.2	91.5±0.0	85.0±0.2	90.4±0.1	69.8±0.0	76.7±0.0	12	2	$0.385 {\pm} 0.019$	
MLP [200]	73.2±0.3	80.3±0.1	84.8±0.1	90.3±0.2	77.1±0.0	85.6±0.0	91.0±0.4	76.1±3.0	0.263±0.008	
DeepFM [15]	73.6±0.2	80.4±0.1	86.1±0.2	91.7±0.1	76.9±0.0	83.4±0.0			0.260 ± 0.006	
DeepGBM [70]	78.0±0.4	84.1±0.1	84.6±0.3	90.8±0.1	74.5±0.0	83.0±0.0		-	0.856 ± 0.065	
RLN [72]	73.2±0.4	80.1±0.4	81.0 ± 1.6	75.9±8.2	71.8±0.2	79.4±0.2	77.2±1.5	92.0±0.9	$0.348 {\pm} 0.013$	
TabNet [5]	81.0 ± 0.1	90.0 ± 0.1	85.4±0.2	91.1±0.1	76.5±1.3	84.9 ± 1.4	93.1±0.2	99.4±0.0	$0.346 {\pm} 0.007$	
VIME [88]	72.7±0.0	79.2 ± 0.0	84.8±0.2	90.5±0.2	76.9±0.2	85.5 ± 0.1	90.9 ± 0.1	82.9±0.7	$0.275 {\pm} 0.007$	
TabTransformer [98]	73.3±0.1	80.1±0.2	85.2±0.2	90.6±0.2	73.8±0.0	81.9±0.0	76.5±0.3	72.9±2.3	$0.451 {\pm} 0.014$	
NODE [6]	79.8±0.2	87.5±0.2	85.6±0.3	91.1±0.2	76.9±0.1	85.4±0.1	89.9±0.1	98.7±0.0	0.276±0.005	
Net-DNF [57]	82.6±0.4	91.5±0.2	85.7±0.2	91.3±0.1	76.6±0.1	85.1±0.1	94.2±0.1	99.1±0.0	1.4	
STG [201]	73.1±0.1	80.0 ± 0.1	85.4±0.1	90.9±0.1	73.9±0.1	81.9±0.1	81.8±0.3	96.2±0.0	0.285 ± 0.006	
NAM [202]	73.3±0.1	80.7±0.3	83.4±0.1	86.6±0.1	53.9±0.6	55.0 ± 1.2		in the second	0.725 ± 0.022	
SAINT [9]	82.1±0.3	90.7±0.2	86.1±0.3	91.6±0.2	79.8±0.0	88.3±0.0	96.3±0.1	99.8±0.0	$0.226 {\pm} 0.004$	

[Borisov et al. 2022]

Models								
Robustness Methods	Fairness Methods	Tabular Tree-Based	Supervised Baselines					
DORO (Chi^2, CVar) DRO (Chi^2, CVar) MWLD Group DRO	LFR Inprocessing (ExpGrad) Postprocessing	XGBoost LightGBM GBM Random Forest	L2 Log. Reg. SVM MLP					

Hyperparameter/Architecture Grid Search

Х

Х

Datasets (incl. 2 sensitive attributes)

 \rightarrow 317k total training iterations

Datasets

Dataset	Label	Sens.	n	d	Smallest Test Subgroup
ACS Income*	High/Low Income	Race, Sex	499,350	20	18,134
ACS PubCov*	Public Ins.	Race, Sex	379,430	19	14,689
BRFSS*	Diabetes	Race, Sex	175,745	28	1,133
LARC	At-Risk (Grade)	URM Status, Sex	169,032	26	8,377
Adult	High/Low Income	Race, Sex	48,845	14	518
COMPAS	Recidivism	Race, Sex	7,215	10	57
Comm. & Crime	Elevated Crime	Income Lvl, Race	1,994	113	36
German Credit	Credit Risk	Age, Sex	1,000	22	11

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Example: Experiment Results (Group DRO, BRFSS)



Example: Experiment Results (Group DRO, BRFSS)



Tree Models Match Robustness Methods





Tree Models Match Robustness Methods



Metrics: Does what we measure matter?



Model Performance Metrics: One Size Does Not Fit All



Model Performance Metrics: One Size Does Not Fit All



Trees are Robust to Model Selection Effects





Trees are Robust to Model Selection Effects



Trees are Less Sensitive to Choice of Hyperparameters





Trees are Less Sensitive to Choice of Hyperparameters



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Tree-based models (XGBoost, LightGBM, etc.) are **surprisingly strong** subgroup robustness baselines.

These models are **cheaper to train**, **less sensitive to hyperparameters**, and **less sensitive to the model selection** metric.







Future Directions

This finding is specific to **MLP-based models**, which are the exclusive (tabular) model evaluated in the robustness works we sought to benchmark.

 \rightarrow Does shifting away from MLPs close the gap with trees?

This may be an artifact of well-known relationship between in-distribution and out-of-distribution accuracy (Miller et al. 2021).

 \rightarrow How can we make neural architectures more tree-like (or adopt differentiable techniques for tree training to use robust learning) to take advantage of this near-linear empirical relationship?

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Subgroup Robustness Grows on Trees: An Empirical Baseline Investigation

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Robust Sparse Mean Estimation via Sum-of-Squares

Sushrut Karmalkar University of Wisconsin-Madison

Joint work with: Ilias Diakonikolas Daniel Kane Ankit Pensia Thanasis Pittas

[COLT'2022]

Goal: Signal recovery in the presence of **arbitrary, adversarial** corruptions.

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An estimator is **robust**, if it is able to estimate the signal, even in the presence of these corruptions.

Given: Samples from a distribution that is adversarially shifted in TV.

Recover: Signal when you know some properties of the inlier distribution

Fraction of Corruptions (ϵ): As **large** as possible.

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Sample complexity: As **small** as possible for the given ϵ .

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Sample complexity: As **small** as possible for the given ϵ .

Runtime: As **small** as possible, as a function of the input size.

Fraction of Corruptions (ϵ)

Sample complexity

Runtime

Fraction of Corruptions (ϵ)

Sample complexity

Classical Robust Statistics [Tukey'60, Huber'64].

Runtime

Fraction of Corruptions (ϵ)

Sample complexity

Runtime

Algorithmic Robust Statistics

[Diakonikolas-Kane-Kamath-Li-Moitra-Stewart'16, Lai-Rao-Vempala'16]

Mean Estimation

Mean Estimation

Given: poly(*d*) samples drawn from \mathscr{D} on \mathbb{R}^d with mean μ . **Recover:** $\hat{\mu}$ such that $\|\hat{\mu} - \mu\|_2$ is small.



Mean Estimation

Given: poly(*d*) samples drawn from \mathscr{D} on \mathbb{R}^d with mean μ . **Recover:** $\hat{\mu}$ such that $\|\hat{\mu} - \mu\|_2$ is small.



Need \mathcal{D} to be structured for the **robust** setting - typically Gaussian, Log-concave etc.

Sparse Mean Estimation

Given: poly(k, log(d)) samples, drawn from \mathscr{D} with mean μ , where μ is *k*-sparse.

Recover: $\hat{\mu}$ such that $\|\hat{\mu} - \mu\|_2$ is small.



Outlier Model

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Robust Sparse Mean Estimation

Given: ϵ -corrupted poly(k, log(d)) size sample set, inliers drawn from \mathcal{D} on \mathbb{R}^d with a *k*-sparse mean μ .

Recover: $\hat{\mu}$ such that $\|\hat{\mu} - \mu\|_2$ is small.



Goal: Non-robust vs robust

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Find an algorithm achieving fastest rate of convergence.

Goal: Non-robust vs robust



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Question: Is there an algorithm in the **sparse** setting which can achieve nearoptimal guarantees with **bounded**, **unknown** covariance?

Known Covariance Setting






















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We give nearly matching Statistical Query lower bound suggesting that $\Omega(k^4)$ samples are necessary.

Setting:

1. The inlier distribution has its first t moments "certifiably" bounded by O(1).*

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- Runs in time $poly((nd)^t)$.
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We give nearly matching Statistical Query lower bound suggesting that this is the optimal guarantee possible.

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*We also need the first t \log(d) moments bounded by O(1).
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Sushrut Karmalkar

• Algorithms for robust sparse mean estimation when the covariance is unknown.

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Questions?

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Thank You